TR19_139_141

17. Programmed calculation of grid-geographic coordinate transformation.

D.E. Leaman

All standard maps produced in Tasmania display metre grid coordinates based on Australian map grid zone 55 with false origin 500 000 mE and 10 000 000 mS of the true origin of the zone. Unfortunately many maps, especially those in preparation for publication, do not show geographic coordinates. As such information is required for various geophysical reductions, notably the latitude correction in gravity surveys, it was found necessary to produce an alternative to the awkward and laborious calculations based on U.S. Department of the Army tables for the conversion. Such tables are too precise for most geophysical purposes, an accuracy of about one second being adequate.

METHOD OF CALCULATION BY TABLES

(1) Relate coordinates to true origin of zone by subtraction of false origin factors.

(2) Determine apparent latitude from northing using equation (table) I.

(3) Adjust apparent latitude using equations VII and VIII by allowing for the eastward component of the coordinate from the origin.

(4) Using the apparent latitude refer to equations IX and X.

(5) Adjust terms IX and X for eastward component from the origin.

(6) Add the corrected factors IX and X to the central longitude of the zone to give true longitude of the point.

This method is standard for the conversion on a transverse Mercator projection and the calculation is wholly inverse, *i.e.* a value of latitude is presumed and then adjusted throughout. For example, Table I has been formed by substitution of values of latitude. There is no simple rearrangement possible so that input of grid values might yield latitude directly.

REVISED METHOD OF CALCULATION

Calculation of longitude is straightforward once the apparent latitude has been determined.

$$i.e.\ \Delta\lambda = \frac{\sec\phi}{N\sin 1''}.x' - \frac{\sec\phi(N/_R + 2\tan^2\phi)}{6N^3\sin 1''}.x'^3$$

$$+ \frac{\sec\phi(5 + 28\tan^2\phi + 24\tan^4\phi)}{120N^5\sin 1''}.x'^5$$

where $\Delta\lambda$ is the difference in longitude from the central meridian of the zone

ϕ is apparent latitude (based on table I)

N is radius of curvature in the prime vertical

$$\left[= \frac{a}{(1 - e^2\sin^2\phi)^{\frac{1}{2}}} = R(1 + e'^2\cos^2\phi)\right]$$

e is eccentricity of spheroid

e' is apparent eccentricity

R is radius of curvature in the meridian

x' is true easting.

The third term may be ignored when the precision required is of the order of one second.

Thus determination of latitude remains the key problem. Table I, on which the latitude calculation is based, is formed from a summation of sine functions for which there is no inverse. This problem has been overcome, within the required limits of accuracy, by using the forward calculation as presented in the tables and dividing the state into four zones:

39°30' - 40°45'	(1)
40°45' - 42°15'	(2)
42°15' - 43°45'	(3)
43°45' - 45°00'	(4)

Within each a special function is produced, which closely approximates function I, but which uses the known grid value:

$$\text{apparent latitude} = k.\sin\left(\frac{GN}{10^6}\right)$$

where k is a factor for each zone and GN is the false northing.

The factor k is chosen such that the maximum error in each zone is less than one second.

A similar process must be used for functions VII, and VIII. However function VIII does not vary significantly across the latitudes of Tasmania and a value of 35.0" has been assumed (35/3600°). The maximum error introduced by this assumption is 0.3 seconds at the zone boundaries. Function VII has been generated by equivalence based on the following system for each zone.

$$\text{VII} \equiv [BV + (\phi_A - \phi_{BV})/3600.k']/3600 \text{ degrees}$$

where BV is the value of VII at the base latitude for each zone (ϕ_{BV})

ϕ_A is apparent latitude

k' is a factor for each zone such that the error is less than 0.3 seconds.

Functions VII and VIII and their equivalents are not especially sensitive to variations in k' and the principal source of error in the entire conversion lies with the equivalent function for I. Indeed if machine capacity were available the number of zones should be increased to eight or ten.

The values of k, k' are tabulated below:

Zone	k	k'
39°30' - 40°45'	518.12	0.02084
40°45' - 42°15'	518.105	0.02181
42°15' - 43°45'	518.092	0.02285
43°45' - 45°00'	518.083	0.02380

PROGRAMME

The programme (10A1) is written for the Wang 700B programmable calculator and is in five parts:

(1) Introductory titling (MARK Π)

(2) Coordinate transformation (MARK O)

  (*a*) Data input; E, N.

  (*b*) Calculation of I equivalent (apparent latitude) after selection of appropriate zone.

(c) Calculation of easting components x', x'^2, x'^3, x'^4.
(d) Calculation of VII equivalent.
(e) Calculation of VIII equivalent.
(f) Adjustment of I by VII x'^2, VIII x'^4.
(g) Direct computation of $\Delta\lambda$ from ϕ_A.

(3) Rearrangement of latitude, longitude values to degrees, minutes and seconds; print out.
(4) Trigonometric sub-routines.
(5) Sub title.

The operation of the programme is based on only three flag searches.

SEARCH Π : Main title.
SEARCH O : Data input.
SEARCH $1/_x$: Sub title.
SEARCH O must be repeated for each pair of coordinates.

Verify number of programme : 8444

SUMMARY

The calculation programmed here is a compromise between machine capacity and accuracy. Fortunately an accuracy of only one second was desired. The calculation is very rapid.

The accuracy achieved varies according to the position of the point within each subdivision and is always better than one half second. Due to rounding during printing this may be apparently exceeded on some occasions. An alternative would be to print the result as decimal degrees.

The actual precision is therefore about 30 m in latitude, and 20 m in longitude.

EXAMPLE OF PRINT-OUT

TASMANIA: transformation of coordinates
GRID TO GEOGRAPHIC

GRID E	GRID N	LATITUDE			LONGITUDE			(METRE GRID)
232752	5589856	39	47	57	143	52	42	
371444	5348352	42		20	145	26	51	
576585	5209152	43	16		147	56	36	

accuracy of calculation approx one second.

[14 March 1974]