UR1984_53 1/16

1984/53. Probabilistic slope stability analysis using Rosenbleuth's method.

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Abstract

The application of Rosenbleuth's method to probabilistic slope stability analysis is described. Comments are made on the variability of input data. Worked examples and statistical tables are given in the Appendices.

INTRODUCTION

The purpose of this report is to introduce readers to the application of Rosenbleuth's method to probabilistic slope stability analysis. It is assumed that readers are experienced in carrying out slope stability analysis on a programmable calculator or a computer. Whether or not you are familiar with slope stability analysis you are referred to Moon (1984*b*) to which this report is complementary. No prior knowledge of probabilistic methods is assumed.

This report is the second of a series of four (the others are Moon, 1984*b*, 1984*c*, and 1984*d*) on various aspects of slope stability work in Tasmania. The Master of Science thesis (Moon, 1984*a*) is complementary to this series of reports.

BASIC CONCEPTS

A good introduction to the understanding and application of probabilistic concepts to slope design is given by Morriss (1984). Although he deals with rock slope design, much of the paper is directly relevant to the analysis of soil slopes.

The primary reason for using a statistical treatment of data for the determination of the probability of the failure of a slope is the *variability* or *uncertainty* in almost every parameter affecting a slope's stability.

When carrying out slope stability analysis several different approaches can be used, including:

(1) analysis of the slope using the mean values (or best estimate) of the variables influencing the analysis, with application of a safety factor requirement to the result;

(2) analysis of the worst possible combination of variable values, with application of a safety factor requirement to the result;

(3) analysis using measured (or best estimate of) variabilities and reporting of the result in terms of probability of failure (or, in the case of back analysis, a probability distribution of a variable).

Neither of the first two approaches tells us what degree of conservatism is being used. For example, is a safety factor of 1.2 adequate? When nothing is known about the potential *variation* in safety factor in the analysis due to *variation* in the variables used to compute the result, the choice of a safety factor becomes meaningless.

The third approach, of analysing the slope using the measured variabilities, overcomes this problem and the result of such analyses reflects the confidence you have in the input parameters.

Another way of looking at the probabilistic approach is to compare it with sensitivity analysis carried out deterministically. Probabilistic analysis may be regarded as simultaneous sensitivity analysis for several variables.

Rosenbleuth's method is the only method of analysis discussed in this report because it is relatively easy to use. Other methods are discussed by Morriss (1984).

ROSENBLEUTH'S METHOD

Rosenbleuth's method is a technique whereby individual variables (which have distributions of values) are replaced by point estimates at fixed distances of one standard deviation either side of the mean value (Rosenbleuth, 1975). The method's application to slope stability analysis has been developed by Harr (McMahon, 1982) and by Morriss (1984). Chowdhury (personal communication) has found that it gives comparable results to other methods of probabilistic slope stability analysis.

HOW TO USE

Approach

Descriptions and worked examples of Rosenbleuth's method are given in the references. The approach adopted here has been to work through two example problems. The computation sheets for the examples are given in Appendix 1.

The examples given here involve two and three variables. When readers have successfully worked through these examples the problems given by Morriss (1984) with two variables, and McMahon (1982) with three variables, should also be tackled. McMahon's notes illustrate how several variables can be handled. Two variables involve four computations, three variables involve eight computations, and n variables involve 2^n computations.

Finally, in order to become adequately familiar with Rosenbleuth's method, it is suggested that readers use the method to help solve some of the slope stability analysis problems given in Appendix 3 of Moon (1984*b*).

Bishop's simplified method of analysis has been used for all the calculations reported here. The particular program used was written for a Hewlett Packard 41C programmable calculator and is described in Moon (1984*b*).

Example problems

The two example problems are based on the analysis of Bovills Slip (Appendix 1). The problems are:

(A) Back analyse for residual friction angle (ϕ'_r) assuming the variations in cohesion (C') and pore pressure ratio (r_u) given below.

(B) Assuming the slip has not recently failed determine the probability of failure using the strength data given below.

Input data

The initial input data and calculations using the means of the variable data are given on the first computation sheet. The data used for the two problems are given in Table 1.

Table 1. INPUT DATA FOR EXAMPLE PROBLEMS

Problem A: find probability distribution for ϕ'_r given:

parameter	*mean*	*standard deviation*	+	-	*Implied 95% confidence interval*
C'_r(kPa)	3	1	4	2	1 to 5
r_u	0.44	0.02	0.46	0.42	0.40 to 0.48

Problem B: find probability of failure given:

parameter	*mean*	*standard deviation*	+	-	*Implied 95% confidence interval*
C'_r	2	1	3	1	0 to 4
ϕ'_r	21	1.5	22.5	19.5	18 to 24
r_u	0.44	0.03	0.47	0.41	0.38 to 0.50

Notes: The symbols + and - refer to the mean plus the standard deviation and the mean minus the standard deviation respectively.

Comments on the variability of data are given later in this report and discussed further in Moon (1984*a* and 1984*c*).

Calculations

The calculations for the two example problems are given in Appendix 1. These computation sheets should be self-explanatory. In considering the further questions at the end of Problem B it is not necessary to recalculate the whole probability distribution for each change of cohesion. It is sufficiently accurate to assume the same standard deviation and to check the variation of the mean safety factor with the mean cohesion.

The statistical tables required for determining the areas under normal distribution curves are given in Appendix 2. Rosenbleuth's method can be used to handle other probability distributions (see References).

A method for calculating the mean and standard deviation of the probability distributions using an HP-41C calculator is given in Appendix 3.

COMMENTS ON THE VARIABILITY OF DATA

Assessing the quality of the input data is the most important part of any stability analysis. An example of input data review is given in Moon (1984*a*).

Assuming the slope profile has been measured the critical inputs where decisions have to be made are given in Table 2 (extract from a standard computation sheet).

Table 2. CRITICAL DATA IN STABILITY ANALYSIS

Calculator store	*Item*
06	unit weight in kPa (gamma)
07	cohesion in kPa (C')
08	friction angle (phi or ϕ')
09	pore pressure ratio (Ru)
10	Y co-ordinate of slip circle, relates to depth of slip (Ycircle)

The unit weight can be determined by measuring the volume and weight of undisturbed samples. It is also possible to roughly estimate the unit weight of a soil if the moisture content is known. This involves the assumption that the soil is saturated (no air voids) and the soil particle density is known, or can be guessed.

For example;

if the moisture content is 30% (m = 0.30), assume soil is saturated and that the soil particle density (Gs) is 2.7 g/cm³, then the bulk density (γ_b) is given by

$$\gamma_b = \frac{Gs(1 + m)}{1 + mGs} \quad g/cm^3$$

$$= 1.939 \ g/cm^3$$

and the unit weight is

$$1.939 \times 9.81$$

$$= 19.0 \ kN/m^3$$

In fact, most stability analyses are insensitive to small changes in unit weight and the accurate determination of this input parameter is relatively unimportant.

The depth of the slip (or the likely depth of a slip on an unfailed slope) is sometimes roughly known or can be guessed from local knowledge. If it is not known the sensitivity of the analysis to varying this parameter should be investigated. It is the writer's experience that stability analysis is relatively insensitive to variations of this parameter within reasonable limits.

The remaining critical parameters (cohesion, friction angle, and pore pressure ratio) are the most important. Slope stability analysis of the landslips commonly occurring on natural slopes in Tasmania is sensitive to all of these parameters. In order to gain greater knowledge of these parameters and confidence in assessing their variability, further study of Tasmanian landslips is required. An approach to such a study, involving a combination of field work, laboratory work, and back analysis is outlined in Moon (1984*c*). Back analysis of existing landslips is a useful tool in determining critical parameters but care should be taken to avoid misleading or unreliable results (Leroueil and Tavenas, 1981).

REFERENCES

* LEROUEIL, S.; TAVENAS, F. 1981. Pitfalls of back-analysis. *Proceedings of the Tenth International Conference of Soil Mechanics and Foundation Engineering, Stockholm.* 1:185-190.

* McMAHON, B.K. 1982. Unpublished notes on Rosenbleuth's method, presented at the Australian Mineral Foundation Workshop Course No. 187/82 on Rock Mechanics in Open Pit Mines.

MOON, A.T. 1984*a*. *Investigation of Bovills Landslip, near Devonport, Tasmania.* M.Sc. thesis. University of Tasmania : Hobart.

MOON, A.T. 1984*b*. Bishop's simplified slope stability analysis with a Hewlett-Packard 41C calculator. *Unpubl.Rep.Dep.Mines Tasm.* 1984/51.

MOON, A.T. 1984*c*. Slope stability assessments in Tasmania - ideas for future work. *Unpubl.Rep.Dep.Mines Tasm.* 1984/55.

MOON, A.T. 1984*d*. Preliminary ideas on the development of guidelines for consultants undertaking slope stability assessments. *Unpubl.Rep. Dep.Mines Tasm.* 1984/56.

* MORRISS, P. 1984. Understanding and applying probabilistic concepts to rock slope design. *Fourth Australia-New Zealand Conference on Geomechanics, Perth.* 2:623-630.

* ROSENBLEUTH, E. 1975. Point estimates for probability moments. *Proceedings of the National Academy of Science, USA.* 10:3812-3814.

[31 July 1984]

* Copies of these papers are held by the Supervising Geologist, Engineering Geology Section.

APPENDIX 1

Worked examples of Rosenbleuth's method of probabilistic slope stability analysis

SLOPE STABILITY ANALYSIS - COMPUTATIONS SHEET 1 OF 5...

NAME OF SLIP ....BOVILLS.......... MAP REF. 4497 54411......

ANALYSIS BY A.T.M.
DATE..12.7.1984...
CHECKED BY.........

INITIAL INPUT			SHAPE *					
STORE	ITEM		STORE	ITEM		STORE	ITEM	
00	X L.H.S.	2·5 m	20	Y_1	6·5	31	Y_{12}	
01	Y L.H.S.	5 m	21	Y_2	9	32	Y_{13}	
02	X R.H.S.	40 m	22	Y_3	10	33	Y_{14}	
03	Y R.H.S.	14 m	23	Y_4	11·5	34	Y_{15}	
04	SLICES	5	24	Y_5	13·5	35	Y_{16}	
06	GAMMA	20 kN/m³	25	Y_6		36	Y_{17}	
07	C	3 kPa	26	Y_7		37	Y_{18}	
08	PHI	? degrees	27	Y_8		38	Y_{19}	
09	ru	0·44	28	Y_9		39	Y_{20}	
10	Y CIRCLE	50 m	29	Y_{10}				
11	F_1	1	30	Y_{11}				

* Only enter Nº specified

CALCULATIONS

PROBLEM 'A' (MEAN)

Given ..F = 1..
(assume C = 3)
Find....Ø....

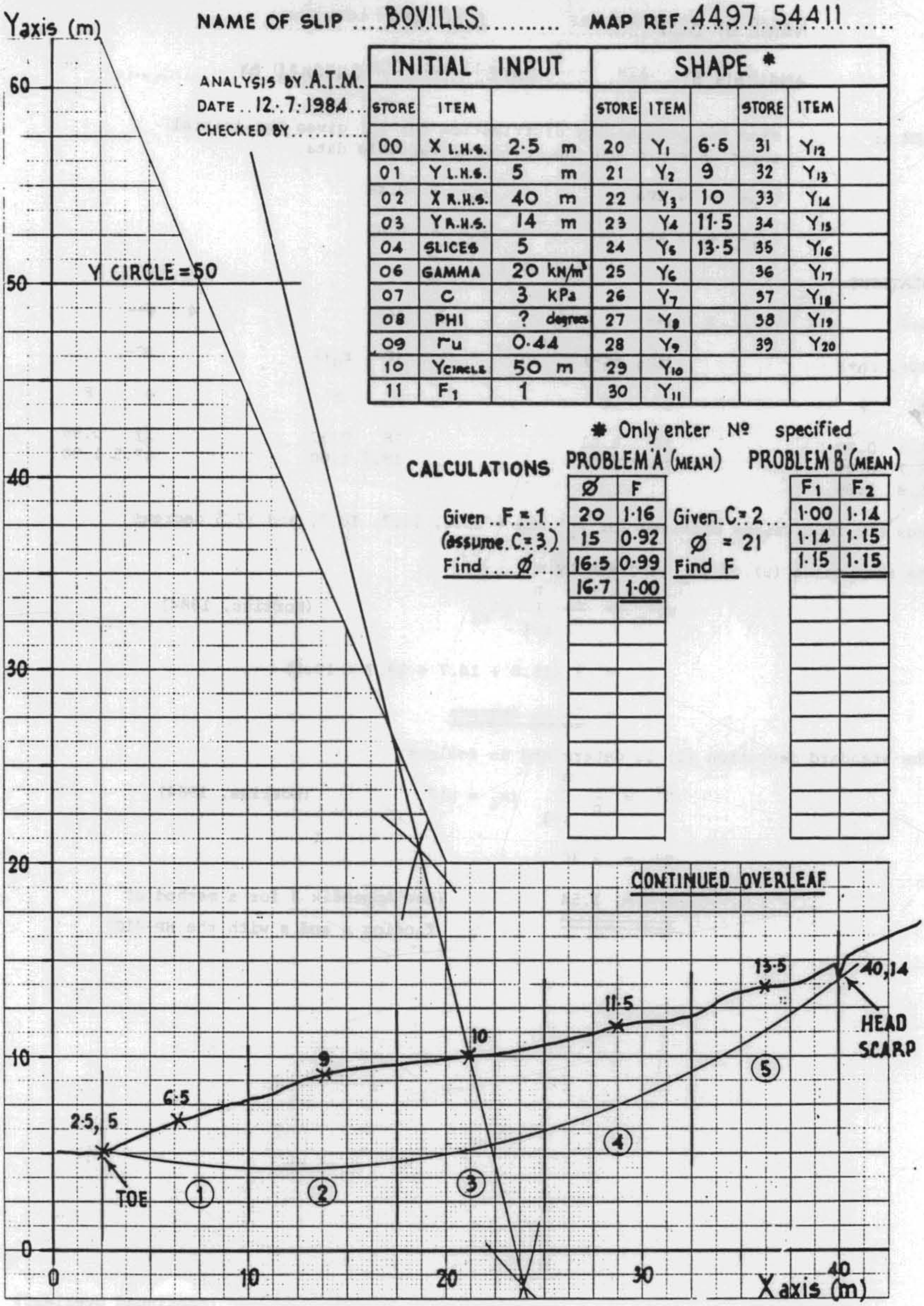
Ø	F
20	1·16
15	0·92
16·5	0·99
16·7	1·00

PROBLEM 'B' (MEAN)

Given C = 2...
Ø = 21
Find F

F_1	F_2
1·00	1·14
1·14	1·15
1·15	1·15

CONTINUED OVERLEAF



5 cm

SLOPE STABILITY ANALYSIS - COMPUTATIONS SHEET 2 OF 5

NAME OF SLIP BOVILLS MAP REF. 4497 54411

ANALYSIS BY ATM DATE 13 July 1984 CHECKED BY

PROBLEM A; Find the probability distribution for ϕ_r' given the initial input as shown and the following variable data,

$C'_r+ = 4$ kPa $r_u+ = 0.46$

$C'_r- = 2$ kPa $r_u- = 0.42$

CALCULATIONS:

1 ϕ++		2 ϕ+-		3 ϕ-+		4 ϕ--	
(C+, r_u+)		(C+, r_u-)		(C-, r_u+)		(C-, r_u-)	
ϕ	F	ϕ	F	ϕ	F	ϕ	F
15.5	0.99	15	1.01	18	0.97	17	0.98
15.7	0.99	14.7	1.00	18.7	1.00	17.5	1.00
15.8	1.00						

Thus the four values obtained for ϕ_r' are = 15.8, 14.7, 18.7, and 17.5 degrees

The mean value (μ) of ϕ'_r is given by =

$$\mu\phi'_r = \frac{1}{n}\sum_{i=1}^{n}\phi'_{ri} \quad \text{(Morriss, 1984)}$$

$$= \tfrac{1}{4}\,(15.8 + 14.7 + 18.7 + 17.5)$$

$$= \underline{16.68 \text{ degrees}}$$

The standard deviation (S) is determined as follows:

$$S^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i = \mu)^2 \quad \text{(Morriss, 1984)}$$

$$S^2 = 2.36$$

$$S = 1.54$$ (see Appendix 3 for a method of finding μ and s with the HP-41C)

(continued overleaf)

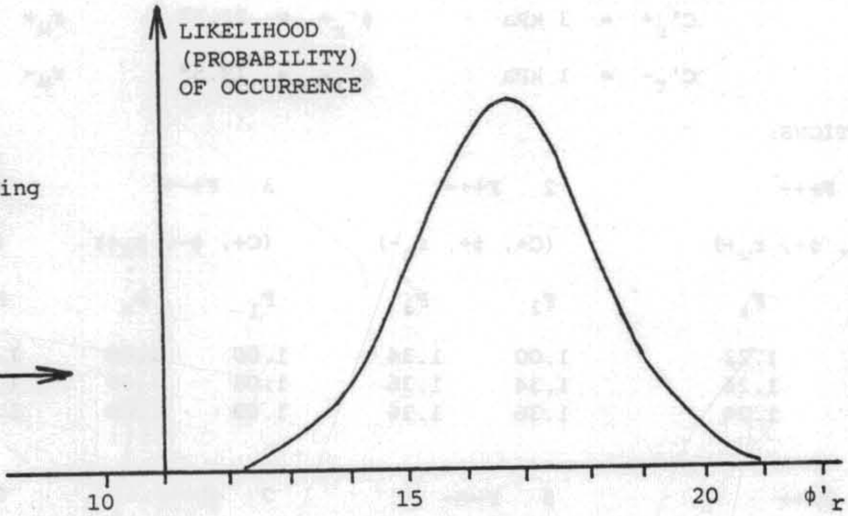
SLOPE STABILITY ANALYSIS - COMPUTATIONS SHEET 3 OF 5

NAME OF SLIP BOVILLS MAP REF. 4497 54411

ANALYSIS BY ATM DATE 13 July 1984 CHECKED BY

PROBLEM A (continued)

The probability distribution for ϕ'_r looks something like this



QUESTION: What is the probability of ϕ'_r being less than 15°?

$$z_{15} = \frac{15 - 16.68}{1.54}$$

(see Morriss, 1984 and tables in Appendix 2)

$$= -1.0909$$

From tables F(z) = 0.138

Thus there is a 13.8% probability of ϕ'_r being less than 15°

Similarly what is the probability of ϕ'_r being less than 20°

$$z_{20} = \frac{20 - 16.68}{1.54} = 2.16$$

$$F(z) = 0.985$$

Thus there is a 98.5% probability of ϕ'_r being less than 20°

5 cm

SLOPE STABILITY ANALYSIS - COMPUTATIONS SHEET 4 OF 5

NAME OF SLIP BOVILLS MAP REF. 4497 54411

ANALYSIS BY ATM DATE 13 July 1984 CHECKED BY

PROBLEM B: Assume the slip has not recently failed. Find the probability of failure given the initial input and the following variable data.

$C'_r+ = 3$ kPa	$\phi'_r+ = 22.5°$	$r_u+ = 0.47$
$C'_r- = 1$ kPa	$\phi'_r- = 19.5°$	$r_u- = 0.41$

CALCULATIONS:

1 F+++ ($C+$, $\phi+$, r_u+)

F_1	F_2
1.00	1.22
1.22	1.24
1.24	1.24

2 F++- ($C+$, $\phi+$, r_u-)

F_1	F_2
1.00	1.34
1.34	1.36
1.36	1.36

3 F+-+ ($C+$, $\phi-$, r_u+)

F_1	F_2
1.00	1.08
1.08	1.09
1.09	1.09

4 F+-- ($C+$, $\phi-$, r_u-)

F_1	F_2
1.00	1.18
1.18	1.20
1.20	1.20

5 F-++

F_1	F_2
1.00	1.09
1.09	1.10
1.10	1.10

6 F-+-

F_1	F_2
1.00	1.21
1.21	1.22
1.22	1.22

7 F--+

F_1	F_2
1.00	0.95
0.95	0.95

8 F---

F_1	F_2
1.00	1.05
1.05	1.06
1.06	1.06

Thus the eight values of F are: 1.24, 1.36, 1.09, 1.20, 1.10, 1.22, 0.95, 1.06

Thus the mean value of F = 1.15 (see Problem A)

and the standard deviation = 0.12

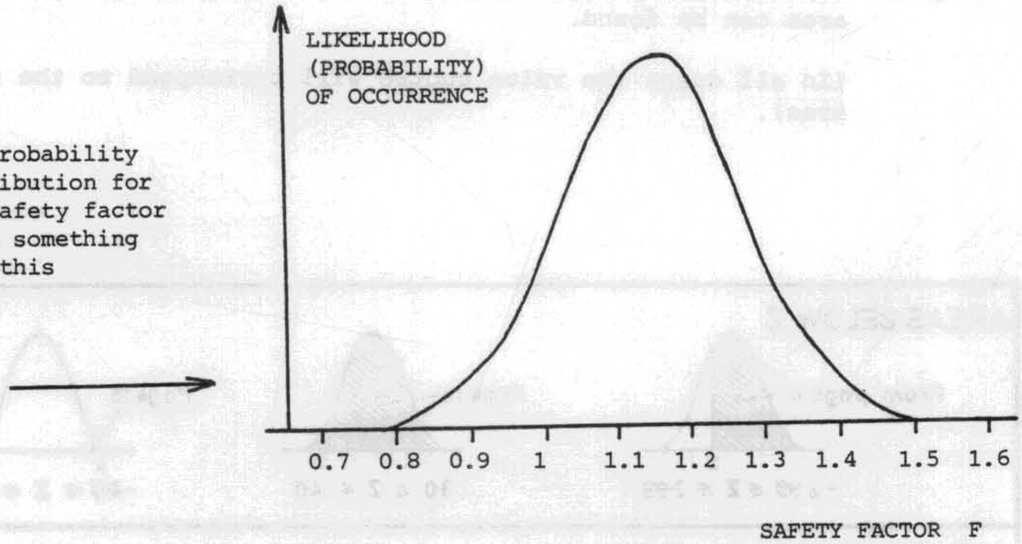
SLOPE STABILITY ANALYSIS - COMPUTATIONS SHEET 5 OF 5

NAME OF SLIP BOVILLS MAP REF. 4497 54411

ANALYSIS BY ATM DATE 13 July 1984 CHECKED BY

PROBLEM B (continued)

The probability distribution for the safety factor looks something like this



QUESTION: What is the probability of the safety factor F being less than 1

$$Z_1 = \frac{1 - 1.15}{0.12} = -1.25$$

From tables F(z) = 0.1056

Thus there is a 10.6% probability of failure

FURTHER QUESTIONS:

What is the probability of failure if the mean cohesion is 1 kPa or 3 kPa or 4 kPa, etc.

(Use the same standard deviation).

Plot a graph of mean cohesion against probability of failure.

5 cm

APPENDIX 2

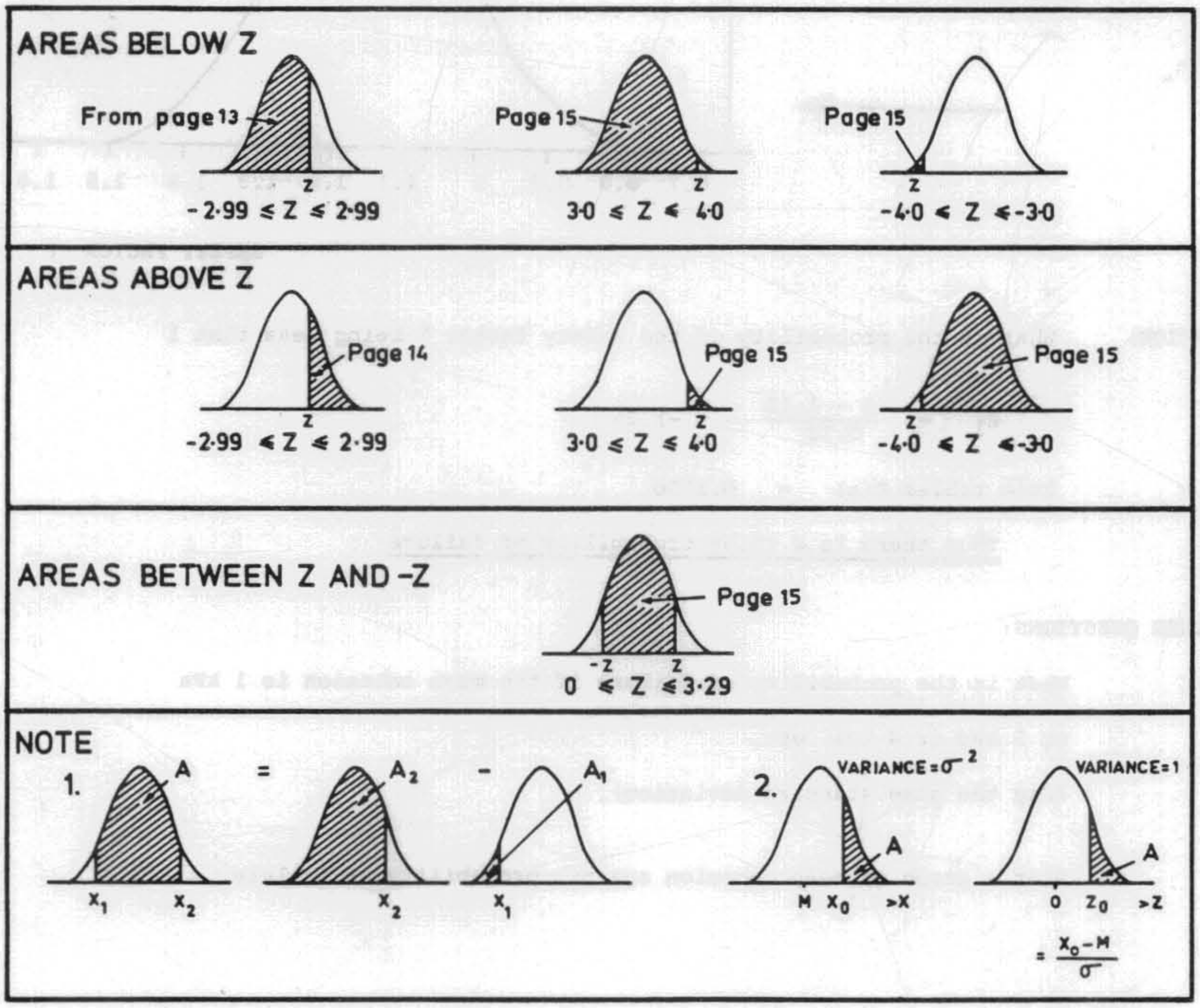
Statistical tables required for determining the areas under normal distribution curves

Determining areas under normal distribution curves using Table A

Step 1: given a value of x and the values of M and σ^2 to define the particular normal distribution, compute $z = (x-M)/\sigma$.

Step 2: use the appropriate diagram below to specify the page where the area can be found.

(In all cases the value quoted will correspond to the shaded area).



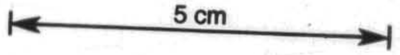


TABLE A - the standard normal distribution (Areas below z)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986

TABLE A - the standard normal distribution (Areas above z)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
-2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
-2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
-2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
-2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
-2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
-2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
-2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
-2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
-2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
-1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
-1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
-1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
-1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
-1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
-1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
-1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
-1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
-1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
-1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
-0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
-0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
-0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
-0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
-0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
-0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
-0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
-0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
-0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014

TABLE A - the standard normal distribution

3. Areas between z and -z for z in the range 0.00 to 3.29

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0080	.0160	.0239	.0319	.0399	.0478	.0558	.0638	.0717
0.1	.0797	.0876	.0955	.1034	.1113	.1192	.1271	.1350	.1428	.1507
0.2	.1585	.1663	.1741	.1819	.1897	.1974	.2051	.2128	.2205	.2282
0.3	.2358	.2434	.2510	.2586	.2661	.2737	.2812	.2886	.2961	.3035
0.4	.3108	.3182	.3255	.3328	.3401	.3473	.3545	.3616	.3688	.3759
0.5	.3829	.3899	.3969	.4039	.4108	.4177	.4245	.4313	.4381	.4448
0.6	.4515	.4581	.4647	.4713	.4778	.4843	.4907	.4971	.5035	.5098
0.7	.5161	.5223	.5285	.5346	.5407	.5467	.5527	.5587	.5646	.5705
0.8	.5763	.5821	.5878	.5935	.5991	.6047	.6102	.6157	.6211	.6265
0.9	.6319	.6372	.6424	.6476	.6528	6579	.6629	.6680	.6729	.6778
1.0	.6827	.6875	.6923	.6970	.7017	.7063	.7109	.7154	.7199	.7243
1.1	.7287	.7330	.7373	.7415	.7457	.7499	.7540	.7580	.7620	.7660
1.2	.7699	.7737	.7775	.7813	.7850	.7887	.7923	.7959	.7995	.8029
1.3	.8064	.8098	.8132	.8165	.8198	.8230	.8262	.8293	.8324	.8355
1.4	.8385	.8415	.8444	.8473	.8501	.8529	.8557	.8584	.8611	.8638
1.5	.8664	.8690	.8715	.8740	.8764	.8789	.8812	.8836	.8859	.8882
1.6	.8904	.8926	.8948	.8969	.8990	.9011	.9031	.9051	.9070	.9090
1.7	.9109	.9127	.9146	.9164	.9181	.9199	.9216	.9233	.9249	.9265
1.8	.9281	.9297	.9312	.9328	.9342	.9357	.9371	.9385	.9399	.9412
1.9	.9426	.9439	.9451	.9464	.9476	.9488	.9500	.9512	.9523	.9534
2.0	.9545	.9556	.9566	.9576	.9586	.9596	.9606	.9615	.9625	.9634
2.1	.9643	.9651	.9660	.9668	.9676	.9684	.9692	.9700	.9707	.9715
2.2	.9722	.9729	.9736	.9743	.9749	.9756	.9762	.9768	.9774	.9780
2.3	.9786	.9791	.9797	.9802	.9807	.9812	.9817	.9822	.9827	.9832
2.4	.9836	.9840	.9845	.9849	.9853	.9857	.9861	.9865	.9869	.9872
2.5	.9876	.9879	.9883	.9886	.9889	.9892	.9895	.9898	.9901	.9904
2.6	.9907	.9909	.9912	.9915	.9917	.9920	.9922	.9924	.9926	.9929
2.7	.9931	.9933	.9935	.9937	.9939	.9940	.9942	.9944	.9946	.9947
2.8	.9949	.9950	.9952	.9953	.9955	.9956	.9958	.9959	.9960	.9961
2.9	.9963	.9964	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972
3.0	.9973	.9974	.9975	.9976	.9976	.9977	.9978	.9979	.9979	.9980
3.1	.9981	.9981	.9982	.9983	.9983	.9984	.9984	.9985	.9985	.9986
3.2	.9986	.9987	.9987	.9988	.9988	.9988	.9989	.9989	.9990	.9790

4. Areas in the tails of the distribution

Areas below z

3.1	.99903	-4.0	.00003
3.2	.99931	-3.9	.00005
3.3	.99952	-3.8	.00007
3.4	.99966	-3.7	.00011
3.5	.99977	-3.6	.00016
3.6	.99984	-3.5	.00023
3.7	.99989	-3.4	.00034
3.8	.99993	-3.3	.00048
3.9	.99995	-3.2	.00069
4.0	.99997	-3.1	.00097
		-3.0	.0013

Areas above z

3.0	.0013	-4.0	.99997
3.1	.00097	-3.9	.99995
3.2	.00069	-3.8	.99993
3.3	.00048	-3.7	.99989
3.4	.00034	-3.6	.99984
3.5	.00023	-3.5	.99977
3.6	.00016	-3.4	.99966
3.7	.00011	-3.3	.99952
3.8	.00007	-3.2	.99931
3.9	.00005	-3.1	.99903
4.0	.00003	-3.0	.9987

APPENDIX 3

A method for calculating the mean and standard deviation of probability distributions using an HP-41C calculator

When using Rosenbleuth's method it is necessary to calculate the mean and standard deviation of the probability distributions (see Morriss, 1984 and second computation sheet of Appendix 1). It is possible to calculate these two parameters by making use of the statistical registers of the HP-41C using the following procedure:

(1) SWITCH CALCULATOR ON

(2) IF 'USER' IS DISPLAYED PRESS THE USER KEY (ENSURE CALCULATOR IS NOT IN USER MODE).

(3) PRESS YELLOW KEY.

(4) PRESS $x \gtrless y$ key (CLΣ)
THIS CLEARS THE STATISTICAL REGISTERS.

(5) ENTER DATA INTO STATISTICAL REGISTERS.
IN THE EXAMPLE GIVEN THE KEYSTROKES ARE:

15.8
Σ+
14.7
Σ+
18.7
Σ+
17.5
Σ+

(6) TO DETERMINE MEAN TAKE THE FOLLOWING STEPS:

RCL 11
RCL 16
÷

THE MEAN IS THEN DISPLAYED.

(7) TO DETERMINE THE STANDARD DEVIATION TAKE THE FOLLOWING STEPS:

RCL 12
RCL 16
÷
RCL 11
RCL 16
÷
x^2 (PRESS YELLOW KEY, THEN $\sqrt{x}$ KEY)
−
$\sqrt{2}$

NOTES: If you wish to carry out stability analysis you need to put the calculator back into USER mode.

Operating the statistical registers will alter the contents of STORE 11 (the safety factor). You may want to re-enter 1 into this store before continuing with your analysis.